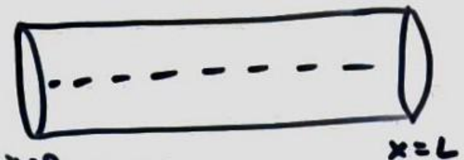
1/ Σωλήνας (αυλός) με ανοικτά άκρα.



Για το κύμα μετατόπισης:

$$\xi(x,t) = (B\sin kx + C\cos kx)\cos(\omega t+\varphi) \quad (1)$$

x=0: $\left.\frac{\partial \xi}{\partial x}\right|_{x=0} = 0 \xrightarrow{(1)} \left.(Bk\cos kx - Ck\sin kx)\cos(\omega t+\varphi)\right|_{x=0} = 0 \rightarrow$

$\rightarrow (Bk\cos 0 - Ck\sin 0)\cos(\omega t+\varphi) = 0 \rightarrow Bk\cos(\omega t+\varphi) = 0 \xrightarrow[k\neq 0]{\forall t}$

$\rightarrow \boxed{B=0}$ Sol. $\xi(x,t) = C\cos kx\cos(\omega t+\varphi)$ (2)

x=L: $\left.\frac{\partial \xi}{\partial x}\right|_{x=L} = 0 \xrightarrow{(2)} -Ck\sin kL\cos(\omega t+\varphi) = 0 \xrightarrow[C,k\neq 0]{\forall t} \sin kL = 0 \rightarrow$

$\rightarrow kL = n\pi \rightarrow \boxed{k_n = \frac{n\pi}{L}}$ $n=1,2,\ldots$

• Για το κύμα πίεσης: $P(x,t) = (B\sin kx + C\cos kx)\cos(\omega t+\varphi)$ (1)

x=0: $P_{(0,t)} = 0 \xrightarrow{(1)} (B\sin 0 + C\cos 0)\cos(\omega t+\varphi) = 0 \xrightarrow{\forall t} \underline{C=0}$

Sol. $P(x,t) = B\sin kx\cos(\omega t+\varphi)$ (2)

x=L: $P_{(L,t)} = 0 \xrightarrow{(2)} B\sin kL\cos(\omega t+\varphi) = 0 \xrightarrow[B\neq 0]{\forall t} \sin kL = 0 \rightarrow kL = n\pi \rightarrow$

$\rightarrow \boxed{k_n = \frac{n\pi}{L}}$ $n=1,2,\ldots$

$$\lambda_n = \frac{2\pi}{k_n} = \frac{2\pi}{\frac{n\pi}{L}} \rightarrow \boxed{\lambda_n = \frac{2L}{n}} \sim \boxed{L = n\frac{\lambda_n}{2}}$$

$$\omega_n = v k_n \rightarrow \omega_n = v\frac{n\pi}{L} \rightarrow 2\pi f_n = v\frac{n\pi}{L} \rightarrow \boxed{f_n = n\frac{v}{2L}}$$

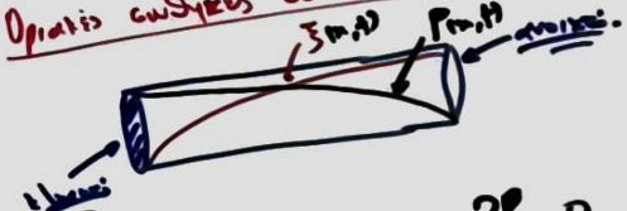
$v = \alpha\sqrt{T}$

Σχηματική αναπαράσταση.

1ος Κ.Τ.Τ. Για $n=1$: $L = \frac{\lambda}{2}$.

2ος Κ.Τ.Τ.: Για $n=2$: $L = \lambda_2$.

Οριακές συνθήκες σε σωλήνα αέρα.



• Κλειστό άκρο: $\xi = 0$ ή $\frac{\partial P}{\partial x} = 0$

• Ανοικτό άκρο: $\frac{\partial \xi}{\partial x} = 0$ ή $P=0$ γιατί $P=P_0$

4) Σωλήνας (αυλός) με κλειστό & ανοικτό άκρο

Για το κύμα μετατόπισης:

$$\xi(x,t) = (B\sin kx + C\cos kx)\cos(\omega t+\varphi) \quad (1)$$

$x=0$: $\xi(0,t) = 0 \xrightarrow{(1)} (B\sin 0 + C\cos 0)\cos(\omega t+\varphi) = 0 \xrightarrow{\forall t}$ $\underline{C=0}$

Άρα $\xi(x,t) = B\sin kx\cos(\omega t+\varphi)$ (2)

$x=L$: $\left.\dfrac{\partial\xi}{\partial x}\right|_{x=L} = 0 \xrightarrow{(2)} Ck\cos kL\cos(\omega t+\varphi) = 0 \xrightarrow[C,k\neq 0]{\forall t} \cos kL = 0 \rightarrow kL = (2n-1)\dfrac{\pi}{2} \rightarrow \boxed{k_n = (2n-1)\dfrac{\pi}{2L}}$ $n=1,2,\dots$

• Για το κύμα πίεσης:

$$P(x,t) = (B\sin kx + C\cos kx)\cos(\omega t+\varphi) \quad (1)$$

$x=0$: $\left.\dfrac{\partial P}{\partial x}\right|_{x=0} = 0 \xrightarrow{(1)} (kB\cos 0 - kC\sin 0)\cos(\omega t+\varphi) = 0 \xrightarrow{\forall t} \underline{B=0}$

Άρα $P(x,t) = C\cos kx\cos(\omega t+\varphi)$ (2)

$x=L$: $P(L,t) = 0 \xrightarrow{(2)} C\cos kL\cos(\omega t+\varphi) = 0 \xrightarrow[C\neq 0]{\forall t} \cos kL = 0 \rightarrow kL = (2n-1)\dfrac{\pi}{2} \rightarrow \boxed{k_n = (2n-1)\dfrac{\pi}{2L}}$ $n=1,2,\dots$

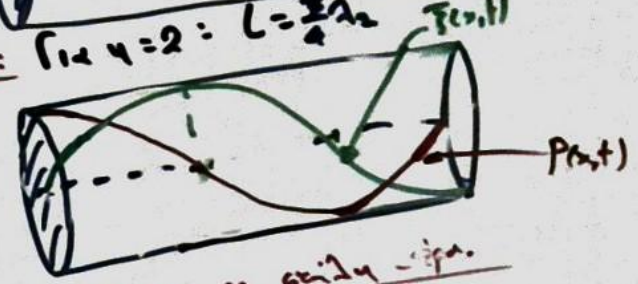
$$\lambda_n = \frac{2\pi}{k_n} = \frac{2\pi}{(2n-1)\frac{\pi}{2L}} \rightarrow \boxed{\lambda_n = \frac{4L}{2n-1}} \sim \boxed{L = (2n-1)\frac{\lambda_n}{4}}$$

$$\omega_n = \underset{\alpha\sqrt{T}}{\underset{\parallel}{\upsilon}} k_n \rightarrow \omega_n = \upsilon(2n-1)\frac{\pi}{2L} \rightarrow 2\pi f_n = \upsilon(2n-1)\frac{\pi}{2L} \rightarrow \boxed{f_n = (2n-1)\frac{\upsilon}{4L}}$$

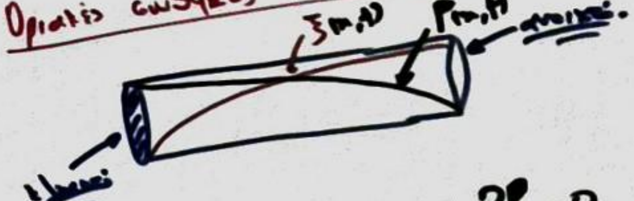
Σχηματική αναπαράσταση.

1ος κ.τ.τ. Για $n=1$: $L = \dfrac{\lambda_1}{4}$.

2ος κ.τ.τ: Για $n=2$: $L = \dfrac{3}{4}\lambda_2$

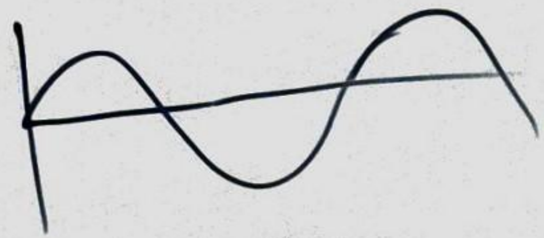


Οριακές συνθήκες σε στήλη αέρα.



• Κλειστό άκρο: $\xi = 0$ ή $\dfrac{\partial P}{\partial x} = 0$

• Ανοικτό άκρο: $\dfrac{\partial \xi}{\partial x} = 0$ ή $P = 0$ γιατί $P = P_0$



Θ.2 24/7/2010

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{q^2}\frac{\partial^2 p}{\partial t^2}$$

$p = P - P_0$

A) $q \rightarrow$ ταχύτητα ήχου στον αέρα

Για: π.χ. πίεση, θερμοκρασία

$q = \alpha\sqrt{T} \simeq 340\ m/s$

B)

$$p(x,t) = (A\cos kx + B\sin kx)\cos\omega t. \quad (1)$$

$A, B, k, \omega = ?$ αν $P(x=0,t=0) = P_0$

Οριακές συνθήκες:

$x=0$: $\left.\frac{\partial p}{\partial x}\right|_{x=0} = 0 \xrightarrow{(1)} \left.(-kA\sin kx + Bk\cos kx)\cos\omega t\right|_{x=0} = 0 \rightarrow$

$\rightarrow (-kA\cancel{\sin 0}^{0} + Bk\cancel{\cos 0}^{1})\cos\omega t = 0 \rightarrow Bk\cos\omega t = 0 \xrightarrow[k\neq 0]{\forall t} \boxed{B=0}$

Δηλ. $p(x,t) = A\cos kx\cos\omega t \quad (2)$

$x=L$: $P(L,t) = 0 \xrightarrow{(2)} A\cos kL\cos\omega t = 0 \xrightarrow[A\neq 0]{\forall t} \cos kL = 0 \rightarrow kL = (2n-1)\frac{\pi}{2} \rightarrow \boxed{k_n = (2n-1)\frac{\pi}{2L}}\ n=1,2,\ldots$

Σχέση διασποράς: $\omega_n = q k_n \rightarrow \boxed{\omega_n = (2n-1)\frac{\pi q}{2L}}$

Αρχική συνθήκη: $P(x=0,t=0) = P_0 \xrightarrow{(2)} A\cos 0\cos 0 = P_0 \rightarrow \boxed{A = P_0}$

Άρα: $\boxed{p(x,t) = P_0\cos\left((2n-1)\frac{\pi}{2L}x\right)\cos\left((2n-1)\frac{\pi q}{2L}t\right)}$

Γ) Αν $L = 0,5m$: $\lambda_3 = ?\ f_3 = ?$ $\lambda_2 = ?\ f_2 = ?$

$\lambda_n = \frac{2\pi}{k_n} = \frac{2\pi}{(2n-1)\frac{\pi}{2L}} \rightarrow \boxed{\lambda_n = \frac{4L}{2n-1}}\ n=1,2,\ldots$

$f_n = \frac{\omega_n}{2\pi} = \frac{(2n-1)\frac{\cancel{\pi}q}{2L}}{2\cancel{\pi}} \rightarrow \boxed{f_n = (2n-1)\frac{q}{4L}}$

• Για $n=1$ (θεμελιώδης): $\lambda_1 = 4L = 2m.$ $f_1 = \frac{q}{4L} = \frac{q}{2} = \frac{340}{2} = 170Hz$

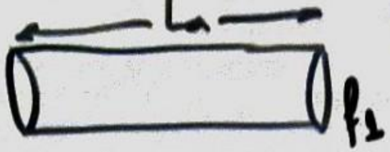
• Για $n=2$ ($1^{η}$ αρμονική): $\lambda_2 = \frac{4L}{3} = \frac{2}{3}m.$ $f_2 = 3\frac{q}{4L} = 3\cdot 170 = 510Hz$

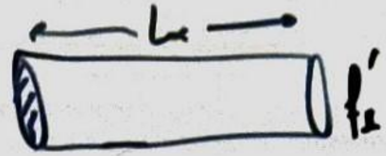
• Για $n=3$ ($2^{η}$ αρμονική): $\lambda_3 = \frac{4L}{5} = \frac{2}{5}m.$ $f_3 = 5\frac{q}{4L} = 5\cdot 170 = 850Hz$

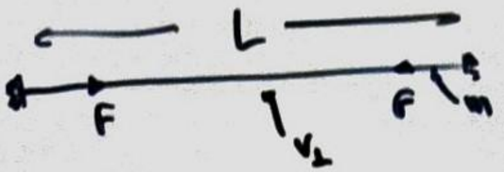
Θ.2 6/7/2013

$L_a = 40cm$
$L = 65cm$
$m = 10g$
$F = 1200N$

$f_1 = ?$
$T = ?$







• Αυλός με ανοιχτά άκρα:

.... $f_1 = n\frac{u_{\eta\chi}}{2L_a} \to \boxed{f_1 = \frac{u_{\eta\chi}}{2L_a}}$ (1)

• Αυλός με κλειστό - ανοιχτό άκρο:

... $f_1' = (2n-1)\frac{u_{\eta\chi}}{4L_a} \xrightarrow{n=1} \boxed{f_1' = \frac{u_{\eta\chi}}{4L_a}}$ (2)

• Παλλόμενη χορδή κιθάρας:

... $v = n\frac{u}{2L}$ (3)

όπου $u = \sqrt{\frac{F}{\mu}}$, $\mu = \frac{m}{L}$ $\Big\} \to u = \sqrt{\frac{FL}{m}}$ (4)

(3) $\overset{(4)}{\sim}$ $v = \frac{n}{2L}\sqrt{\frac{FL}{m}} = \frac{n}{2}\sqrt{\frac{FL}{L^2 m}} \xrightarrow{n=1} \boxed{v_1 = \frac{1}{2}\sqrt{\frac{F}{Lm}}}$ (5)

Επειδή η χορδή συντονίζεται με τον αυλό με κλειστό άκρο ισχύει:

$f_1' = v_1 \xrightarrow{(2,5)} \frac{u_{\eta\chi}}{4L_a} = \frac{1}{2}\sqrt{\frac{F}{Lm}} \to u_{\eta\chi} = 2L_a\sqrt{\frac{F}{Lm}}$ (6)

Άρα: (1) $\overset{(6)}{\sim}$ $f_1 = \frac{\cancel{2L_a}}{\cancel{2L_a}}\sqrt{\frac{F}{Lm}} = \sqrt{\frac{F}{Lm}} = \sqrt{\frac{1200}{0,65\cdot 0,01}} = \sqrt{0,184\cdot 10^6} \to$

$\to f_1 = 0,43\cdot 10^3\ Hz \to \boxed{f_1 = 430\ Hz}$

$u_{\eta\chi} = \alpha\sqrt{T} \to \sqrt{T} = \frac{u_{\eta\chi}}{\alpha} \to T = \frac{u_{\eta\chi}^2}{\alpha^2}$

όπου (6) ~ $u_{\eta\chi} = 2\cdot 0,4\cdot 430 = 0,8\cdot 430 = 344\ \frac{m}{s}$

$\to T = \left(\frac{344}{20,055}\right)^2 = 17,15^2 = 293\ °K.$

$\theta = T - 273 \simeq 20°C.$